Topic 4: Fairness over time in combinatorial optimization

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This project is in combinatorial optimization (CO) with a focus on fairness and fairness over time. We will investigate different concepts of fairness (over time) which are appropriate for CO problems. We plan to analyze the structural properties of the fair solutions and propose solution approaches for fairness over time models related to classical CO problems, ranging from efficiently solvable (e.g. spanning tree or matching problems) to hard ones (e.g. the travelling salesman problem). Depending on the student's background and interests the project offers a wide spectrum of research questions, ranging from easy starting questions to very challenging ones.

A more specific description of the topic, including some references, is given below.

Scientific background. Recently, there has been a growing interest in incorporating fairness in combinatorial optimization (CO). In classical CO problems an aggregated objective function for a set of individual subjects needs to be optimized over a finite (or countably infinite) set of feasible solutions. In many models of real life problems, the individual subjects have individual utility functions which are in conflict with the aggregated objective function. As a result, optimal solutions might lead to discrepancies on the utility values for the individual subjects. Large discrepancies are perceived as unfair and are therefore undesirable. Thus, feasible solutions which optimize the aggregated objective function while maximizing some measure of fairness are of particular interest. If the optimization problems are solved over time as in many models inspired from real life applications, the fairness of the solutions should be also assessed over time. In this context, the design of appropriate measures of fairness over time (FoT) is a relevant and challenging research question. Recently, FoT models related to allocation and transportation problems have been investigated [2, 3].

Aims. We will investigate specific FoT models for classical CO problems such as matching problems, spanning tree problems and variants of the travelling salesman problem. We will introduce different fairness measures and analyze the structural properties of the corresponding fair solutions and the complexity of the resulting optimization problems (FoTPs). A particular goal is the identification of fairness measures leading to tractable FoTPs along with the development of exact, approximate and heuristic solution approaches. Another aspect of research is the price of fairness over time, a concept analogous to the price of fairness introduced in [1].

Methods. From a methodological point of view, this project will involve approaches from different areas of expertise, especially combinatorial optimization, integer programming, algorithmic graph theory, operations research and complexity theory. To gain insight into the structural properties of fair solutions arising in different FoT models we will investigate formulations of the latter which are amenable to known solution approaches in CO, e.g. mixed integer linear programming, dynamic programming or multicriteria optimization. Specific combinatorial properties of fair solutions might lead to tractable versions of FoTPs related to different CO problems and different FoT measures. In particular, we will distinguish between FoTPs where the underlying CO problem is polynomially solvable (for example, the minimum spanning tree problem), and FoTPs where the underlying CO problem is NP-hard (for example, variants of the travelling salesman problem). We will analyze the price of fairness over time in different FoT models aiming at the computation of non-trivial bounds.

References

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[3] A. Lodi, S. Sankaranarayanan and G. Wang (2023),
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