Topic 2: Distribution of point configurations: theory and applications

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Finding a "well-distributed" point set on a given compact space (manifold, even homogeneous manifold) is more difficult than it may seem at first glance. Here the meaning of the phrase "well-distributed" may depend on the actual setting. Common measures for the quality of point distributions are discrepancy (Kolmogorov-Smirnov-statistics), error in numerical integration, packing radius, or covering radius. Even in the seemingly simple case of the sphere $\mathbb{S}^2$ no explicit construction of a point set is known which (provably) has a better discrepancy than uniform random points. One particular aim of this project is to study known point sets, construct new ones, and possibly finally prove that one of them beats uniform random points with respect to one of the quality measures.

Another aim of the project concerns the concept of Poissonian pair correlation (of deterministic point sets), which is a local statistics of point sets defined by

$$\frac{1}{N}\#\left\{i \neq j \in \{1,\ldots,N\}:\ \|x_i - x_j\| \leq \frac{s}{N^{\frac{1}{d}}}\right\},$$

for point sets in a d-dimensional space. For i.i.d. random points the limit for $N \to \infty$ equals Cs^d for a suitable constant C. Deterministic point sets are said to have Poissonian pair correlation if they share this asymptotic behavior. The problems in this part of the project are somewhat diametrical to the problems mentioned before, where points "better than random" were looked for. Now we rather try to establish that certain point sets of interest behave in the same way as random point sets do.

To summarize, the project is concerned with the distribution of finite point sets; on the one hand, we study the existence/construction of point sets which are "more regular" than random point sets (with the aim of using them in applications), while on the other hand we study given point sets of interest (often of arithmetic origin) and try to establish that they behave like random point sets.